year. The quantities used should be rough estimates of what the quantities were throughout the period or epoch. For instance, if you are trying to get the course of the price-level since 1800, you might begin with rough estimates of the quantities for the period 1800–1815—a war period. After that, another set of estimates of the quantities should be used, which might last for ten years. Ten-year terms might be used down to 1875, after which five-year changes might be introduced. These should be still further shortened as you come nearer to the present. For recent years—and especially for the continued use of index numbers into the future—the quantities of every year should be used, and a mean should somehow be drawn between those of every two years. If the geometric mean is antipathetic to you, use the arithmetic, in Bowley's system. The difference between it and the geometric will be but trifling. But, after all, the geometric mean, being used only once in each yearly computation, is by no means beyond the amount of industry we have a right to expect from those who labor with figures. I have no quarrel with Professor Mitchell because, in editing the immense work on the *History of Prices During the War*, he attempted to use Paasche's index number and succeeded in using it only for the first three years, falling into the use of Laspeyres' for the last year, all on the quantities of an intermediate year. He was in a great hurry, and had a right to choose a simple system. But if his work is to become a permanent governmental function—if perchance it should ever be made the guiding principle for Professor Fisher's own scheme of stabilizing the dollar—then we want the very best possible system. Remember then: for the past, use Lowe's index number, and for the present and the future, use Fisher's.

DISCUSSION

By Warren M. Persons

Professor Fisher's analysis of index numbers enables us to use them with greater confidence than we formerly had in the reality of the results. He has shown that they are instruments of precision.

You may have noticed that Professor Fisher has discussed the method of averaging and of weighting index numbers of prices and production without referring explicitly to the *purpose* for which those index numbers were constructed. It seems to me that the methods of averaging and of weighting, as well as the selection of the basic data, all depend upon the purpose to which the index number is to be applied. Moreover, it is evident that Professor Fisher had a specific purpose in mind. His "ideal" index number is an index number that fits into the equation of exchange. He holds that the product of a

price index, P, for any year and the corresponding quantity index, Q, must equal the relative aggregate value, V, of that year with respect to the base period. I agree that this is a legitimate criterion for indices to be used in the equation of exchange, but I am by no means sure that the criterion is one to be applied universally.

As a result of the application of his criteria for a good index number, Professor Fisher recommends as "ideal" the geometric mean between two ratios of weighted aggregates. The weights depend upon a combination of the prices and production in the given year with those in the base year. The speaker, you will notice, referred merely to *two* years in his discussion, although his charts present a comparison of the indices for several years. An index number is not computed merely to compare the index number of one given year with that for the base year but to compare the indices for a series of years with each other. Variable weighting, which Professor Fisher recommends, has the defect that we do not know whether changes in the indices result from changes in prices or production. Also the limitations to dual comparisons imposed by Professor Fisher in his discussion require a new set of weights for every pair of years compared—an endless computing job.

A problem upon which I have been working recently illustrates, first, the necessity, sometimes at least, of securing indices that are comparable each with the other and, second, the confusion that may result from the use of variable weights. I desire to find the correlation between the prices and the volume of production of twelve crops combined. To weight prices by varying production and production by varying prices and then correlate the results is to introduce an unknown element of spurious correlation. I believe that there are better ways of solving my problem than by the use of variable weights—if, indeed, the problem is solvable with variable weights.

Professor Fisher has stated that the arithmetic mean is "biased" upward and the harmonic mean is "biased" downward, without making clear his criterion of bias. The simple arithmetic mean is always larger than the geometric mean, which in turn is larger than the harmonic mean. The *weighted* means, however, are not always arranged in this order. In fact, as Professor A. A. Young has pointed out to me, the arithmetic mean of a number of items is identical with the harmonic mean of those items obtained by using correlative weights. Thus, if $p'_1, p'_2, \cdots \cdots p'_n$ represent the prices of n commodities in any year, and if $q_1, q_2, \cdots \cdots q_n$ represent the quantities of those commodities produced (or exchanged) in the base year, then the

$$\text{weighted arithmetic mean} = \frac{p'_1q_1+p'_2q_2+ \cdots\cdots\cdots +p'_nq_n}{q_1+q_2+ \cdots\cdots\cdots q_n},$$

and the

$$\text{weighted harmonic mean} = \frac{p'_1q_1+p'_2q_2+ \cdots\cdots\cdots +p'_nq_n}{\frac{1}{p'_1}p'_1q_1+\frac{1}{p'_2}p'_2q_2+ \cdots\cdots +\frac{1}{p'_n}p'_nq_n}$$

$$= \frac{p'_1q_1+p'_2q_2+ \cdots\cdots\cdots +p'_nq_n}{q_1+q_2+ \cdots\cdots\cdots q_n}.$$

The weighted arithmetic and harmonic means, with properly chosen weights, therefore, are identical and neither can be called "biased" with respect to the other. When Professor Fisher uses the word "bias," it is not clear to me "with respect to what" the average is biased.

REJOINDER BY IRVING FISHER

First let me thank my critics for their kind words and express my gratification that they agree with me so far as they do. The disagreements which have been expressed are, I believe, in many cases more apparent than real; and I venture to hope that any real disagreements remaining may melt away as the field is threshed over a little more thoroughly.

I am very glad to join heartily with Mr. Walsh in advocating the practical use of what I call the "ideal" formula whenever the necessary data as to quantities for both years are available. It is only when these data are not available—when merely the quantities for the base year are available, that I should propose some other formula as a makeshift.

But Professor Persons and Professor Mitchell are unwilling, at present, to agree with Mr. Walsh and myself on the "ideal" formula as the best for all purposes.

As I understand it, they express doubt rather than disbelief, and make the appeal that the particular *purpose* for which an index number is to be used must surely make a difference.

I quite agree that the purpose of an index number is a very important factor in determining what is the best index number. This is certainly true as to the elements of an index number other than the formula—the character and number of commodities, for instance. But as to the mathematical formula itself, I take a different view.

It is true in the broad field of science, that various types of mathematical averages are best suited for various widely separated purposes.